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A note on heat transfer modelling of Newtonian nanofluids in laminar free convection

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Abstract

The natural convection heat transfer of Newtonian nanofluids in a laminar external boundary-layer is investigated from the integral formalism approach. In particular, this study deals with γ-Al_2O_3/water nanofluids whose Newtonian behaviour was experimentally confirmed for particle volume fractions less than 4% [N. Putra, W. Roetzel, S.K. Das, Natural convection of nano-fluids, Heat and Mass Transfer 39 (2003) 775–784; S.Z. Heris, S.Gh. Etemad, M.N. Esfahany, Experimental investigation of oxide nanofluids laminar flow convective heat transfer, Int. Comm. Heat Mass Transfer 33 (2006) 529–535; R. Prasher, D. Song, J. Wang, Measurements of nanofluid viscosity and its implications for thermal applications, Appl. Phys. Lett. 89 (2006) 133108]. Based on a macroscopic modelling and under the assumption of constant thermophysical nanofluid properties, it is shown that special care has to be exercised in drawing generalized conclusions about the heat transfer enhancement with the use of nanofluids. It has been found that natural convection heat transfer is not solely characterized by the nanofluid effective thermal conductivity and that the sensitivity to the viscosity model used seems undeniable and plays a key role in the heat transfer behaviour.

Keywords: Nanofluids; Natural convection; Heat transfer

1. Introduction

The purpose of this Note is to enhance the discussion on the use of nanofluids with the sole aim of increasing the heat transfer coefficient in convection flows. Until now, it was known that in forced convection [1,2] as well as in mixed convection, using nanofluids could produce a considerable enhancement of the heat transfer coefficient that increased with increasing the nanoparticle volume fraction. One of the major reasons was that nanoparticles enhance heat transfer rate by increasing the thermal conductivity of the resulting nanofluid and incurring thermal dispersion in the flow [3,4]. Consequently, many researches have focused on the way to increase the thermal conductivity parameter by modifying the particle volume fraction, the particle size/shape or the base fluid [5–7]. However, it is worth mentioning that a recent work [8] in forced convection indicates that the assessment of the heat transfer enhancement potential of nanofluid is difficult and closely dependent on the way the nanofluid thermophysical properties are modelled.

Unlike forced convection, there is a striking lack of theoretical and experimental data in natural convection. Furthermore, the conclusions from the few published results in the literature also seem to be controversial. For example for a buoyancy-driven flow in a two-dimensional enclosure, Khanafer et al. [9] have numerically found that the nanofluid heat transfer rate increases with the increase of nanoparticle volume fraction. On the other hand, the experimental study by Putra et al. [10] for a natural convection case of copper and alumina–water nanofluids inside a horizontal differentially heated cylinder has shown an apparently paradoxical behaviour of significant heat transfer deterioration. Wen and Ding [11], using titanium dioxide nanoparticles, have also observed experimentally such deterioration in the natural convective heat transfer.

Because knowledge of nanofluids are still at their early stages, it seems very difficult to have a precise idea on the way the use of nanoparticles acts in natural convection heat trans-

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fer and complementary works are needed. Thus, to remedy this lack of data and to document the natural convection heat transfer, a full model is used for nanofluid applications in external boundary-layer flows. Due to the constant fluid properties assumption used for this model, the present investigation shall be restricted to Newtonian nanofluids. For example, among the few experimental data available in literature, such a Newtonian behaviour was experimentally confirmed for Al_2O_3/water suspensions as well as for CuO/water suspensions [1,10,12]. On the other hand, the non-Newtonian character of some other nanofluids such as the shear thinning behaviour of CNT nanotubes [13] makes the model unsuitable for them. In order to formulate the mathematical modelling foundation, the integral formalism has been derived based on the assumption that distinctive scaling lengths are considered for the dynamical and thermal boundary layer thicknesses, and that the ratio $\Delta = \delta_T/\delta$ of the thermal boundary layer thickness (δ_T) to that of the dynamical one (δ) is dependent only on the Prandtl number [14] and not on the flow regime. The complete theoretical development associated with such an integral approach in the laminar case has been previously presented in details for the (UWT) condition in [15] and for the (UHF) condition in [16,17] for both laminar and turbulent free convection regimes. Therefore, for the sake of space, only the analytical solutions for the relative boundary layer thickness and the convective heat transfer coefficient are presented in this note.

2. Heat transfer general modelling

The physical system consists of a heated vertical plate where laminar natural convection occurs. The theoretical model assumes sufficiently small (realistic) temperature gradients across the boundary layer, so that thermophysical properties of the nanofluid are assumed to be constant except for the density variation in the buoyancy force, which is based on the incompressible fluid Boussinesq approximation. Since the solid particles have reduced dimension (∼40 nm) and they are easily fluidized, these particles can be considered to have a fluid-like behaviour [2,3]. Furthermore, by assuming a local thermal equilibrium state and negligible motion slip between the particles and the continuous phase, the nanofluid can be treated as a common pure fluid. Thus, one may expect that the classic theory for single-phase fluids can be extended to nanofluids. Consequently, the usual integral forms of the boundary-layer momentum and energy conservation equations can be directly extended to nanofluids:

$$\begin{cases} \frac{\partial}{\partial x}\int_0^{\delta_{nf}} U^2\,dy = g\beta_{nf}\int_0^{(\Delta\delta)_{nf}}(T-T_\infty)\,dy \\ \qquad\qquad - \nu_{nf}\left(\frac{\partial U}{\partial y}\right)_{y=0} \\ \frac{\partial}{\partial x}\int_0^{(\Delta\delta)_{nf}}(T-T_\infty)U\,dy = \frac{\nu_{nf}}{Pr_{nf}}\left(\frac{\partial T}{\partial y}\right)_{y=0} \end{cases} \tag{1}$$

Solving analytically system (1) with physically correct fourth order polynomial profiles for flow velocity and temperature across their respective hydrodynamic and thermal boundary layers [14–17] leads to a seventh-order polynomial in terms of $\Delta(Pr)$.

$$\Delta_{nf}^7 - \frac{799}{126}\Delta_{nf}^6 + \frac{225}{14}\Delta_{nf}^5 - \frac{134}{7}\Delta_{nf}^4 + \frac{20}{3}\Delta_{nf}^3 + \frac{\Omega}{Pr_{nf}} = 0$$
$$\text{with} \begin{cases} \Omega_{UHF} = \frac{10}{9} \\ \Omega_{UWT} = \frac{250}{189} \end{cases} \tag{2}$$

For conciseness, the details of the theoretical approach to access the heat transfer coefficient h are not repeated here. Using Fourier's law, the convective coefficient is expressed for both (UHF) and (UWT) surface conditions as follows:

$$h_{nf}(UWT) = \left[\frac{25 g\beta_{nf}\theta_w k_{nf}^4}{378(9\Delta_{nf}-5)\Delta_{nf}^4\nu_{nf}^2 x}\right]^{\frac{1}{4}} \tag{3}$$

for the (UWT) surface condition and

$$h_{nf}(UHF) = \left[\frac{2 g\beta_{nf}\varphi_w k_{nf}^4}{27(9\Delta_{nf}-5)\Delta_{nf}^4\nu_{nf}^2 x}\right]^{\frac{1}{5}} \tag{4}$$

for the (UHF) surface condition.

In order to assess the influence of the particle volume concentration on a reference heat transfer, let build the average Nusselt number along the wall in terms of the base-fluid Grashof number

$$\overline{Nu_{nf}} = \frac{\overline{h_{nf}}L}{k_{bf}} \tag{5}$$

Thus, the average Nusselt number calculation yields

$$\overline{Nu_{nf}} = \frac{4\sqrt{5}}{3\Delta_{nf}}\left[\frac{\beta_r k_r^4}{378\nu_r^2(9\Delta_{nf}-5)}Gr_{bf}\right]^{\frac{1}{4}} \tag{6}$$

for the (UWT) surface condition and

$$\overline{Nu_{nf}^*} = \frac{6}{5}\left[\frac{2\beta_r k_r^4}{27\nu_r^2(9\Delta_{nf}-5)\Delta_{nf}^4}Gr_{bf}^*\right]^{\frac{1}{5}} \tag{7}$$

for the (UHF) surface condition, where Gr_{bf}^* is the modified Grashof number defined as follows

$$Gr_{bf}^* = \frac{g\beta_{nf}\varphi_w L^4}{k_{bf}\nu_{bf}^2} \tag{8}$$

The convective heat transfer performance is called ε and defined as

$$\varepsilon(\%) = 100\left[\frac{X_{nf}}{X_{bf}} - 1\right] \tag{9}$$

where X is equally well the heat transfer parameter $h, \overline{Nu}$ or $\overline{Nu^*}$.

Reporting either Eqs. (3), (4) or (6), (7) in expression (5) and introducing the function

$$f(\Delta) = \frac{9\Delta_{bf}^5 - 5\Delta_{bf}^4}{9\Delta_{nf}^5 - 5\Delta_{nf}^4} \tag{10}$$

yields the following characterization of the ε-function

$$\varepsilon(\%) = 100\left[\left(\frac{\beta_r k_r^4}{\nu_r^2}f(\Delta)\right)^{\alpha} - 1\right] \tag{11}$$

where the subscript "r" refers to the nanofluid/base fluid ratio and the α-constant depends on the surface condition and is either 1/5 for the (UHF) or 1/4 for the (UWT) conditions.

3. Thermophysical properties of the γ-Al_2O_3/H_2O nanofluid

Introducing the particle volume fraction ϕ, the thermophysical properties of the nanofluid, namely the density, heat capacity and volumetric expansion coefficient, have been calculated from nanoparticle and base fluid properties at the ambient temperature using the following classic formulas:

$$\rho_{nf} = (1-\phi)\rho_{bf} + \phi\rho_p \tag{12}$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_{bf} + \phi(\rho\beta)_p \tag{13}$$

$$(C_p)_{nf} = (1-\phi)(C_p)_{bf} + \phi(C_p)_p \tag{14a}$$

Remark. As mentioned in Buongiorno [18], assuming that the nanoparticles and the base fluid are in thermal equilibrium, the nanofluid specific heat should be derived from:

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_{bf} + \phi(\rho C_p)_p \tag{14b}$$

However, several authors [2,18–20] prefer to use the simpler approach Eq. (14a) to qualify the specific heat. To be consistent with them, we also have used this formulation.

The Wasp model has been used for calculating the effective thermal conductivity of the nanofluid [3,4]

$$k_r = \frac{k_{nf}}{k_{bf}} = \frac{k_p + 2k_{fb} - 2\phi(k_{bf} - k_p)}{k_p + 2k_{fb} + \phi(k_{bf} - k_p)} \tag{15}$$

Remark. The Wasp model is for macroscopic systems. This simplified model is commonly used in convection problems and restricted to spherical nanoparticles in solid/liquid mixtures. The reason of its utilization in the present study is twofold: (i) microscopic models are still in the early development stage and the few data in the literature deduced from microscopic models applied to alumina/water nanofluids (see for example R.K. Shukla and V.K. Dhir [21]) indicate that for small temperature gradients, a simplified macroscopic model is largely sufficient to estimate the thermal conductivity, and (ii) predicted thermal conductivity ratios obtained with this macroscopic model have been found to be in good agreement with experimental results of Al_2O_3 nanofluids (see Lee et al. [22]).

To illustrate the fact that the heat transfer performance of the nanofluids is not solely characterized by the effective thermal conductivity, we have considered the two following different models for the viscosity usually mentioned in the literature for oxide particle nanofluids. Model (I) is attributed to Brinkman [23] and is currently used in literature (Xuan and Roetzel [3]; Khanafer et al. [9]; Gosselin and da Silva [24]) for nanofluids flowing under the natural convection regime as

$$\text{Model (I)} \qquad \frac{\mu_{nf}}{\mu_{bf}} = \frac{1}{(1-\phi)^{\frac{5}{2}}} \tag{16}$$

Model (II) is derived from experimental data for alumina–water nanofluids and has recently been proposed by Maïga et al. [2, 20] to provide a better modelling of such nanofluids as

$$\text{Model (II)} \qquad \frac{\mu_{nf}}{\mu_{bf}} = 123\phi^2 + 7.3\phi + 1 \tag{17}$$

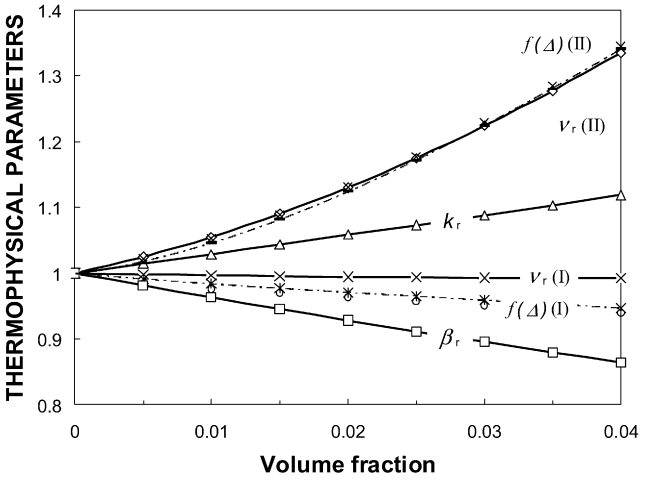

Fig. 1. Thermophysical parameters of the γ-Al_2O_3/H_2O nanofluid.

Variations of the thermal expansion coefficient, the conductivity and the kinematic viscosity are shown in Table 1 and displayed in Fig. 1 for the water-based nanofluid containing γ-Al_2O_3 with average diameter 42 nm initially at 20 °C. Numerically deduced from Eq. (2), the boundary-layer parameter $f(\Delta)$ is also presented.

Several points can be noted from Fig. 1 where the normalized parameters are plotted as a function of the nanoparticle fraction. Results are presented only for fractions up to 4% as no experimental data could be found in the literature concerning the rheological behaviour of the nanofluid above this value. First, one can note that the effective thermal conductivity and volumetric expansion coefficient vary linearly with particle loading under the conditions of the present work. A very significant increase in thermal conductivity is observed, with a conductivity enhancement of ∼12% for the particle volume fraction 4%. These results do agree with those of Eastman et al. [25] for 35 nm average diameter Al_2O_3 particles. A *contrario*, a deterioration of the volumetric expansion coefficient of ∼14% is achieved at the same 4% particle loading. Another major observation is that the behaviour of the normalized dynamic viscosity is highly dependent on the model used. Using model (I) yields a quasi-constant effective dynamic viscosity on the particle loading range while model (II) gives a drastic augmentation of the dynamic viscosity of ∼34% at a 4% particle loading. One will also note that the normalized dynamical parameter $f(\Delta)$ presents an evolution close to that of the dynamic viscosity, due to the fact that $f(\Delta)$ is Prandtl number dependent. It is also worth mentioning that the evolution of $f(\Delta)$ is independent of thermal conditions.

4. Heat transfer results with the γ-Al_2O_3/H_2O nanofluid

The average Nusselt number (6), (7) based on the base-fluid thermal conductivity is plotted in Fig. 2 for the two (UWT) and (UHF) conditions and two Grashof numbers in the laminar regime. To validate the analytical modelling, the classic corre-

Table 1
Physical properties of the γ-Al_2O_3/H_2O nanofluid at 20 °C

	volume fraction (%) of the γ-Al_2O_3/H_2O nanofluid					
	H_2O only	Al_2O_3 only	1	2	3	4
ρ [kg m^{-3}]	998.3	3880	1027.12	1055.93	1084.75	1113.57
C_p [J kg^{-1} K^{-1}]	4182	773	4147.91	4113.82	4079.73	4045.64
k [W m^{-1} K^{-1}]	0.60	36	0.617	0.635	0.653	0.671
k_r	–	–	1.029	1.058	1.088	1.119
β [1 K^{-1}]10^{-6}	206	5	198.41	191.23	184.43	177.99
β_r	–	–	0.963	0.928	0.895	0.864
MODEL (I)						
ν_{I} [m^{-2} s^{-1}]10^{-6}	1.00	–	0.997	0.994	0.993	0.993
ν_r (I)	1.00	–	0.997	0.994	0.993	0.993
Pr (I)	6.96	–	6.88	6.80	6.73	6.66
Δ_{UHF} (I)	0.654	–	0.655	0.656	0.657	0.657
Δ_{UWT} (I)	0.667	–	0.669	0.670	0.670	0.671
MODEL (II)						
ν_{II} [m^{-2} s^{-1}]10^{-6}	1.00	–	1.055	1.130	1.224	1.335
ν_r (II)	1.00	–	1.055	1.130	1.224	1.335
Pr (II)	6.96	–	7.28	7.73	8.29	8.96
Δ_{UHF} (II)	0.654	–	0.651	0.647	0.642	0.637
Δ_{UWT} (II)	0.667	–	0.664	0.659	0.654	0.649

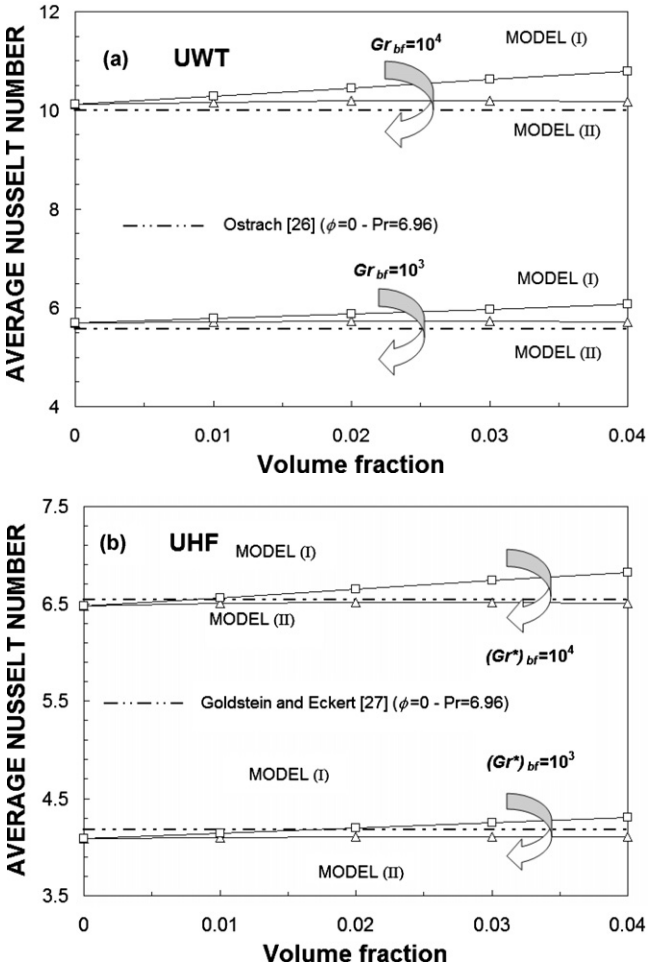

Fig. 2. The average Nusselt number vs the particle loading (a) (*UWT*) condition; (b) (*UHF*) condition.

lations for external boundary-layer natural convection deduced from Ostrach's differential method [26]

$$\overline{Nu}_{\phi=0} = \left[\frac{Pr^2 Gr}{2.43478 + 4.884\sqrt{Pr} + 4.95283Pr}\right]^{0.25} \tag{18}$$

for the (*UWT*) condition and the one attributed to Goldstein and Eckert [27] for the (*UHF*) condition

$$\overline{Nu^*}_{\phi=0} = 0.703(Pr\,Gr^*)^{0.2} \tag{19}$$

are also reported, showing a close agreement for the reference case $\phi = 0$. For both thermal surface conditions considered, the average Nusselt number varies linearly with the volume fraction. This behaviour is similar to that mentioned in [9] and shows a significant heat transfer enhancement inherent to the use of Model (I). The results have also shown that the effective viscosity plays a key role in the natural convection heat transfer, as illustrated by the curves built with Model (II). Indeed, under the present work conditions, one can state that the use of nanoparticles for the purpose of heat transfer enhancement in natural convection with Model (II) seems illusory.

In order to get more details about these conclusions, Fig. 3 presents the evolution of the ε-heat transfer performance versus the particle concentration. The first observation is that for a given viscosity model, a similar trend is found for both thermal surface conditions considered. If Model (I) always predicts an important enhancement of the nanofluid performance, a more contrasted conclusion can be drawn with Model (II). For the latter, a slight enhancement occurs in the particle loading range with a maximum value of about 0.6% for a 2.5% volume fraction followed by a slight trend to a deterioration phenomenon. We recall that due to the lack of experimental data, the study is restricted to volume fractions up to 4%.

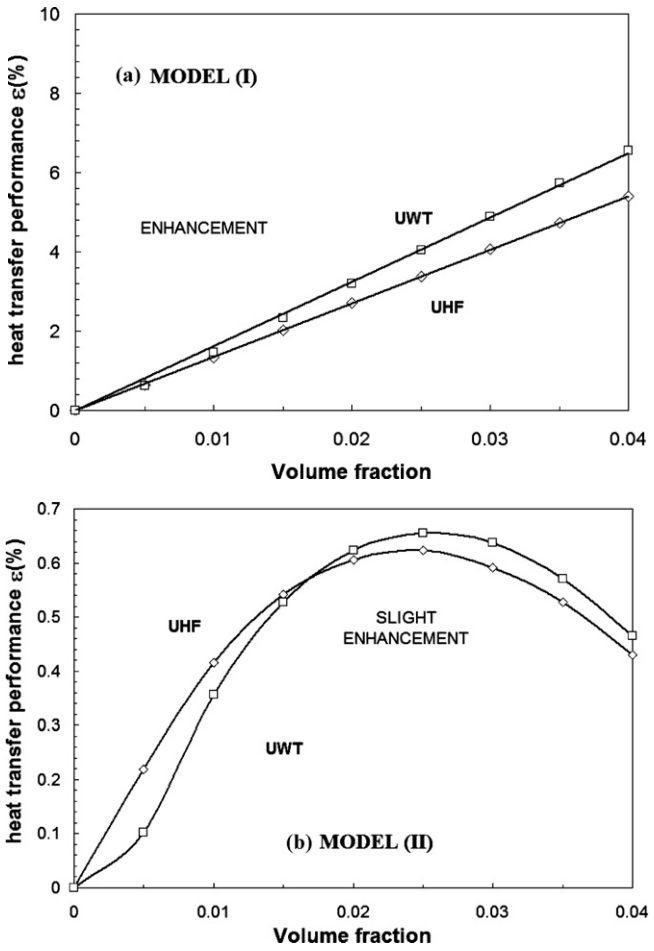

Fig. 3. Heat transfer peformance vs the particle loading (a) Model (I); (b) Model (II)

One will notice that the extrapolation of the curves beyond the 4% volume fraction should yield to a degradation in the heat transfer process as long as the nanofluid behaviour remains Newtonian. As seen in Eq. (10), what qualifies the heat transfer performance in external boundary-layer natural convection is the ratio $\frac{\beta_r k_r^4}{\nu_r^2} f(\Delta)$. Under the present adopted conditions, the following rough approximation $f(\Delta) \approx \nu_r$ can be established from Fig. 1 independently of the thermal viscosity model. With such an approximation, a direct analysis of the new ratio $(\frac{\beta_r k_r^4}{\nu_r})$ seems sufficient to predict the heat transfer performance.

The above numerical results have eloquently shown that the use of Newtonian nanofluids for the purpose of heat transfer enhancement in natural convection situations is not obvious, as such enhancement is dependent not only on nanofluids effective thermal conductivities but also on their viscosities as well. The effect due to the kinematic viscosity seems to be dominant in the natural convective heat transfer in the way that such property will act more or less favorably on the enhancement process according to the model used. Hence, caution should be taken in analyzing results, and further research efforts are, indeed, needed to develop suitable rheological models to predict the effective viscosity of such mixtures.

5. Conclusion

In the present study, the problem of natural convection flow and heat transfer of Newtonian alumina–water nanofluids over a vertical semi-infinite plate has been investigated from a theoretical viewpoint, for a range of nanoparticle volume fractions up to 4%. The analysis is based on a macroscopic modelling and under the assumption of constant thermophysical nanofluid properties. Semi-analytical formulas of heat transfer parameters have been proposed for both the uniform wall temperature (*UWT*) and uniform heat flux (*UHF*) surface thermal conditions. Contrary to the general belief, it has been found that natural convection heat transfer is not solely characterized by the effective thermal conductivity. In fact, in external boundary-layer natural convection, heat transfer is found to be characterized by the parameter group $\frac{\beta_r k_r^4}{\nu_r^2} f(\Delta)$ where $f(\Delta)$ is a boundary-layer parameter function only of the mixture Prandtl number. Our results have shown that in natural convection, special care has to be exercised in drawing generalized conclusions about the heat transfer enhancement, because the sensitivity to the viscosity model used seems undeniable.

Thus, for both the (*UWT*) and (*UHF*) surface thermal conditions, the examination of two viscosity models leads to contrasted conclusions. The use of the Brinkman model for the dynamic viscosity yields a systematic and significant heat transfer enhancement, regardless of the particle concentration. On the other hand, the use of the experimental correlation for the viscosity leads only to a weak enhancement (less than 1%) with a trend to a deterioration phenomenon with increasing the particle concentration in the considered range.

Such contrasted conclusions indicate that further experimental works are obviously needed to establish realistic viscosity models in order to analyze the role of nanoparticles in the natural convective heat transfer process. Even if the present study focused only on the effect of the viscosity, the determination of accurate relations should be extended to other nanofluid properties in order to avoid the resulting ambiguity in drawing conclusions about heat transfer performance, as recently mentioned in forced convection [8].

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